Solvable Criterion for the Contextuality of any Prepare-and-Measure Scenario

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Abstract.
This work aims at offering a mathematical framework that allows one to assess the classicality or non-classicality of any prepare-and-measure scenario, including in particular scenarios consisting of preparing a quantum state out of a given set of quantum states and performing a measurement out of a given set of quantum measurements. The notion of classicality considered here extends Spekkens’ notion of non-contextuality \(^1\) in a strict sense and thus goes one step further in the direction of extending the realm of applicability of this notion of classicality.

It turns out that the question of “does a given prepare-and-measure scenario admit such a generalized non-contextual model?” can be solved in a relatively direct way by verifying a newly introduced unit separability criterion associated to the scenario. Indeed, this criterion admits a reformulation in terms of a vertex enumeration problem. Furthermore, this criterion allows one to infer some strong constraints on the structure of the classical model. In particular, the number of hidden classical states that may describe the quantum system associated to the prepare-and-measure scenario is upper-bounded by the square of the dimension of the Hilbert space. The existence of this bound was an open problem formulated in the language of generalized probabilistic theories in Ref. \(^2\).

In this talk, we would like to describe and motivate the formal definition of the classical model that we consider, present and discuss the results that were derived, and describe the future research directions that would benefit from this framework as a whole. These include most importantly the application of the classicality criterion to modern protocols in quantum computation and in quantum information theory. These applications would aim at contributing to answer the general question of what are the non-classical resources enabling quantum-over-classical advantages in such protocols.

Keywords.
Quantum foundations, non-contextuality, generalized probabilistic theories.

References.